STUDENT NUMBER: ______________________

TEACHER: ______________________________

Founded 1982

THE HILLS GRAMMAR SCHOOL

Trial Higher School Certificate Examination 2015

MATHEMATICS EXTENSION 1

Time Allowed: Two hours (plus five minutes reading time)

Weighting: 40%

Outcomes: H6, H7, H8, H9, HE1, HE2, HE4, HE7, HE9

General Instructions:

- Board-approved calculators may be used
- Attempt all questions
- Start all questions on a new sheet of paper
- The marks for each question are indicated on the examination
- Show all necessary working for Questions 11-14
- The diagrams are not drawn to scale
- A table of standard integrals is provided

Total Marks – 70

Section I Questions 1-10
10 Marks
Allow about 15 minutes for this section

Section II Questions 11-14
60 Marks
Allow about 1 hour and 45 minutes for this section

MCQ	Question 11	Question 12	Question 13	Question 14	TOTAL
/10	/15	/15	/15	/15	/70

Section 1 Multiple Choice (10 Marks)

1 Given that $\tan(\frac{\theta}{2}) = t$, then $\sin\theta$ would be written as:

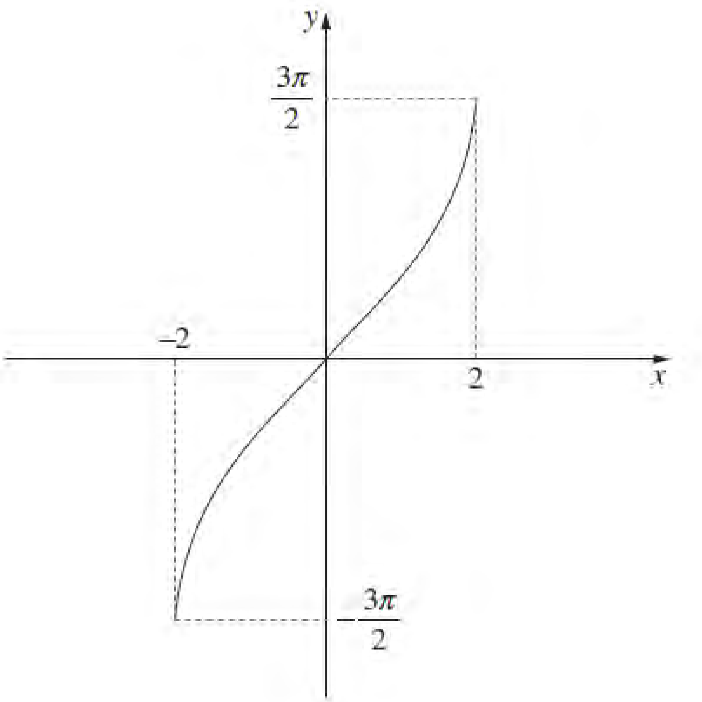
(A) $\frac{2t}{1-t^2}$

(B) $\frac{1-t^2}{1+t^2}$

(C) $\frac{1+t^2}{1-t^2}$

(D) $\frac{2t}{1+t^2}$

2 Which function best describes the following graph:



(A) $y = 3\sin^{-1} 2x$

(B) $y = \frac{3}{2}\sin^{-1} 2x$

(C) $y = 3\sin^{-1} \frac{x}{2}$

(D) $y = \frac{3}{2}\sin^{-1} \frac{x}{2}$

3 Evaluate $\sum_{n=3}^{10} 8+5n$

(A) 283.5

(B) 324

(C) 567

(D) 648

4 The interval AB is divided internally in the ratio 3:1 by the point $P(x, y)$. Given $A(-7, 7)$ and $B(1, -5)$ then the values of x and y are:

(A) $x = -2$ and $y = 2$

(B) $x = 2.5$ and $y = 2$

(C) $x = -1$ and $y = -2$

(D) $x = 1$ and $y = -2$

5 Which expression is the correct factorisation of $x^3 - 27$?

(A) $(x-3)(x^2 - 3x + 9)$

(B) $(x-3)(x^2 - 6x + 9)$

(C) $(x-3)(x^2 + 3x + 9)$

(D) $(x-3)(x^2 + 6x + 9)$

6 The parametric equation of a function is:

$$x = 2t^2,\ y = 4 - t.$$

The Cartesian equation is

(A) $x = 4(2-y)^2$

(B) $x = 2(y-4)^2$

(C) $x = 2(y+4)^2$

(D) $x = 2(4-y)^2$

7 Evaluate $\lim_{x \to 0} \frac{x}{\sin 2x}$:

(A) 0

(B) 0.5

(C) ∞

(D) 2

8 Which expression is equal to $\int \sin^2 3x \, dx$:

(A) $\frac{1}{2}\left(x - \frac{1}{3}\sin 3x\right) + C$

(B) $\frac{1}{2}\left(x + \frac{1}{3}\sin 3x\right) + C$

(C) $\frac{1}{2}\left(x - \frac{1}{6}\sin 6x\right) + C$

(D) $\frac{1}{2}\left(x + \frac{1}{6}\sin 6x\right) + C$

9 A particle is moving in simple harmonic motion with displacement *x*. Its velocity *v* is given by

$$v^2 = 16(9 - x^2)$$

What is the amplitude, *A*, and the period, *T*, of the motion?

(A) $A = 3$ and $T = \frac{\pi}{2}$

(B) $A = 3$ and $T = \frac{\pi}{4}$

(C) $A = 4$ and $T = \frac{\pi}{3}$

(D) $A = 4$ and $T = \frac{2\pi}{3}$

10 The polynomial $P(x) = x^3 + ax^2 + ax + 1$ leaves a remainder of 3 when divided by $(x - 2)$.
The value of *a* is:

(A) 1

(B) -1

(C) -3

(D) 3

END OF SECTION 1

Section 2 **Marks**

BEGIN A NEW BOOKLET
Question 11 (15 marks)

(a) Find $\frac{d^2}{dx^2}e^{x^2}$. **2**

(b) Find k such that $\int_1^k (3-\frac{1}{x^2})dx = 0$. **3**

(c) Use the substitution $u = 1+e^x$ to evaluate $\int_0^{\ln 2} \frac{e^x}{(e^x+1)^2}dx$. **4**

(d) Let $I = \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$.

(i) Find, by integration, the exact value of I. **2**

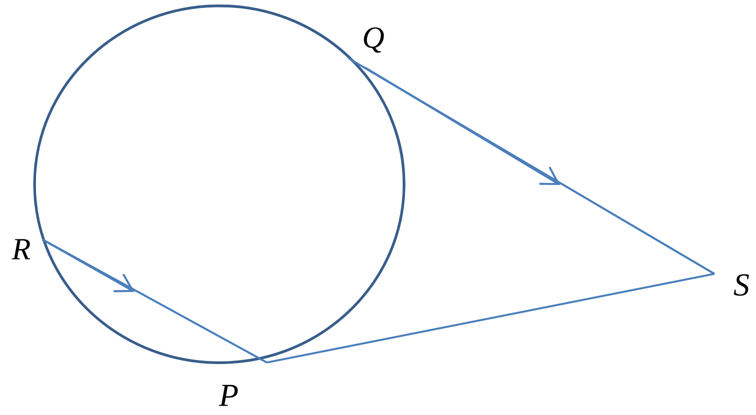
(ii) Use Simpson's rule with 3 function values to approximate I. **2**

(e) i) Show that $e^{x\ln 2} = 2^x$. **1**

ii) Hence find $\frac{d}{dx}2^x$ **1**

Question 12 (15 marks) BEGIN A NEW BOOKLET

(a) In the diagram the points P and Q lie on a circle and the tangents to the circle at P and Q meet at S.
R is a point on the circle so that RP is parallel to QS.



Copy or trace the diagram into your writing book.

i) Explain why ΔPSQ is isosceles, **2**

ii) Show that ΔPQR is isosceles, **2**

iii) Deduce that $QP = QR$. **1**

(b) Detective Angela Baker is called to a murder scene at 3:27a.m. She measures the victim's body temperature at that time to be $27°C$ and one hour later it has dropped to $25°C$. The cooling rate of the body is proportional to the difference between the room temperature $21°C$ and the temperature T , of the body. That is, T satisfies the equation

$$\frac{dT}{dt} = -k(T - 21)$$ where k is a positive constant, and t is the number of hours after 3:27a.m.

(i) Verify that $T = 21 + Ae^{-kt}$ is a solution of this equation, where A is a constant. **1**

(ii) Find the exact values of A and k. **2**

(iii) Assuming that the victim's body temperature was $37°C$ at the time of death, when was the murder committed?
Give your answer to the nearest minute. **2**

Question 12 continued

(c) If $\alpha,\ \beta,$ and γ are the roots of the equation $2x^3 - x^2 - 5x + 6 = 0$ find the value of $\alpha^2 + \beta^2 + \gamma^2$. **2**

(d) Use mathematical induction to show that for all integers $n \geq 1$,

$$\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$ **3**

Question 13 (15 marks) BEGIN A NEW BOOKLET

(a) (i) Prove, using calculus, that the equation $x^3 + 2x + 4 = 0$ has only one real root α. **2**

(ii) Show that $-2 < \alpha < -1$. **1**

(iii) Starting with an initial approximation of $\alpha = -1$, use one application of Newton's method to find a further approximation for α. **2**

(b) A particle is moving in simple harmonic motion along the *x*- axis. Its velocity *v*, at *x*, is given by $v^2 = 24 - 8x - 2x^2$.

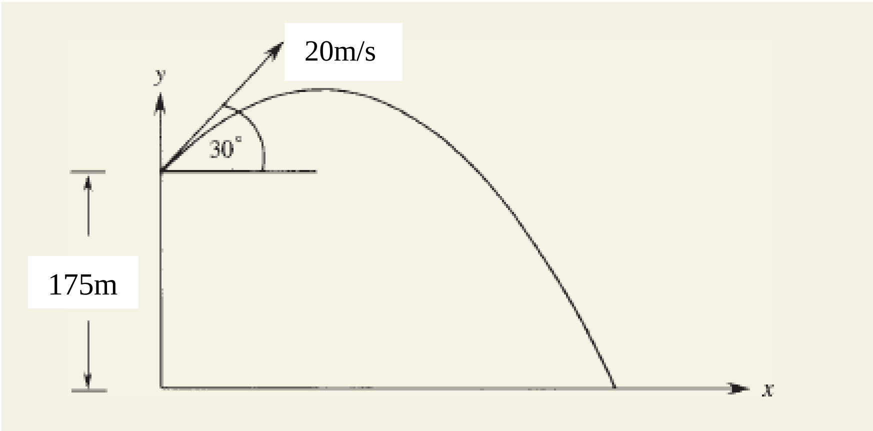
(i) Find all values of *x* for which the particle is at rest. **2**

(ii) Find an expression for the acceleration of the particle, in terms of *x*. **1**

(iii) Find the maximum speed of the particle. **2**

Question 13 continued

(c) A man who is standing on top of a vertical cliff throws a stone into the air at an angle θ to the horizontal. The top of the cliff is 175 metres above a flat sea.



The initial velocity of the stone is $20ms^{-1}$. Acceleration due to gravity is $-10ms^{-2}$.
The path of the stone is given by the parametric equations

$$x = 20t\cos\theta \quad \text{and} \quad y = 20t\sin\theta - 5t^2 + 175$$

The angle of projection of the stone to the horizontal is $30°$.

(i) Find the time it takes for the stone to hit the water. **3**

(ii) Find the speed at which the stone hits the water. **2**

Question 14 (15 marks) BEGIN A NEW BOOKLET

(a) (i) Write $\cos x - \sqrt{3}\sin x$ in the form $R\cos(x+\alpha)$ where $R > 0$ and $0 \le \alpha \le \frac{\pi}{2}$, **2**

(ii) Hence, or otherwise, solve the equation $\cos x - \sqrt{3}\sin x = 1$ for $0 \le x \le 2\pi$. **2**

Question 14 continued

(b) The point $P(2ap, ap^2)$ lies on the parabola $x^2 = 4ay$.

(i) Show that the equation of the tangent to $x^2 = 4ay$ at P is $px - y - ap^2 = 0$. **2**

(ii) The tangent at P cuts the x axis at X. Find the coordinates of X. **1**

(iii) Show that PX is perpendicular to SX, where S is the focus of the parabola. **2**

(iv) A circle is drawn through the points S, X, and P. Show that the coordinates of the centre of the circle are given by

$$C = (ap, \frac{a(1+p^2)}{2}) .$$

Justify your answer. **2**

Question 14 continued

(c) From a point P due south of a vertical tower, the angle of elevation of the top of the tower is $20°$ and from a point Q due east of the tower it is $35°$.
The distance from P to Q is 40 metres.

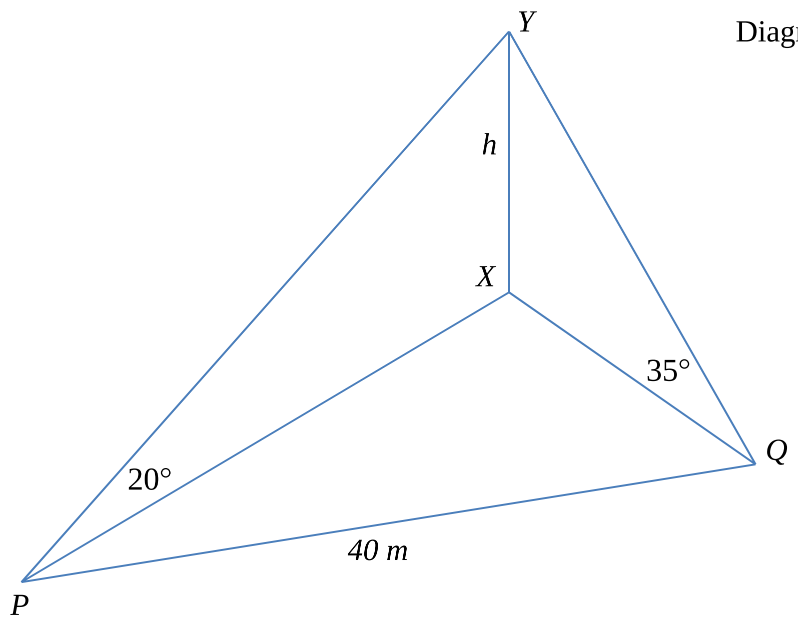


Diagram not to scale

(i) Find an expression for PX in terms of h. **1**

(ii) Find an expression for QX in terms of h. **1**

(iii) Calculate the height of the tower to the nearest metre. **2**

END OF ASSESSMENT TASK

ANSWER SHEET FOR MULTIPLE CHOICE SECTION

Student Exam number:______________________________

Teacher: ___________________________________

1. A ⬭ B ⬭ C ⬭ D ⬭
2. A ⬭ B ⬭ C ⬭ D ⬭
3. A ⬭ B ⬭ C ⬭ D ⬭
4. A ⬭ B ⬭ C ⬭ D ⬭
5. A ⬭ B ⬭ C ⬭ D ⬭
6. A ⬭ B ⬭ C ⬭ D ⬭
7. A ⬭ B ⬭ C ⬭ D ⬭
8. A ⬭ B ⬭ C ⬭ D ⬭
9. A ⬭ B ⬭ C ⬭ D ⬭
10. A ⬭ B ⬭ C ⬭ D ⬭

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Section 1 Multiple Choice (10 Marks)

1 Given that $\tan(\frac{\theta}{2}) = t$, then $\sin\theta$ would be written as:

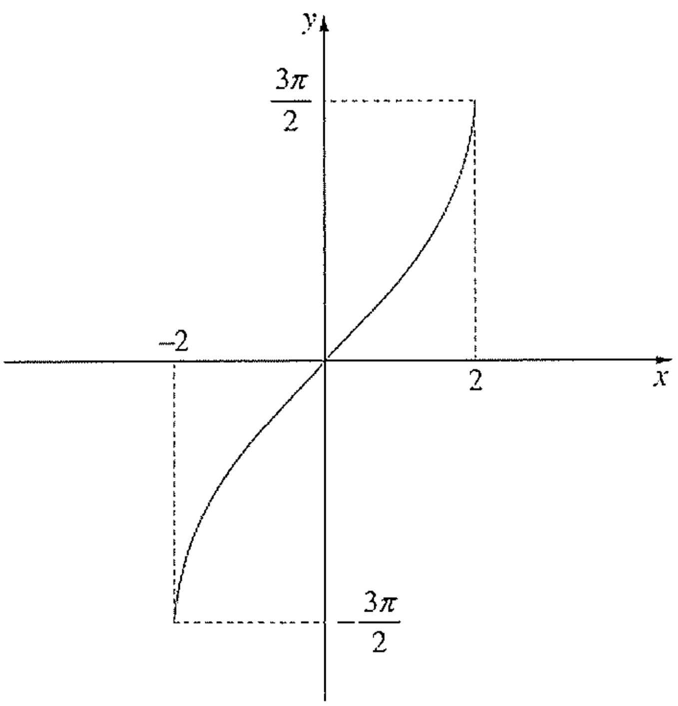
(A) $\frac{2t}{1-t^2}$

(B) $\frac{1-t^2}{1+t^2}$

(C) $\frac{1+t^2}{1-t^2}$

(D) $\frac{2t}{1+t^2}$

2 Which function best describes the following graph:



(A) $y = 3\sin^{-1} 2x$

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(C) $y = 3\sin^{-1} \frac{x}{2}$

(D) $y = \frac{3}{2}\sin^{-1} \frac{x}{2}$

3 Evaluate $\sum_{n=3}^{10} 8+5n$

(A) 283.5 (B) 324

(C) 567 (D) 648

4 The interval AB is divided internally in the ratio 3:1 by the point $P(x,y)$. Given $A(-7,7)$ and $B(1,-5)$ then the values of x and y are:

(A) $x=-2$ and $y=2$

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(C) $x=-1$ and $y=-2$

(D) $x=1$ and $y=-2$

5 Which expression is the correct factorisation of x^3-27?

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(B) $(x-3)(x^2-6x+9)$

(C) $(x-3)(x^2+3x+9)$

(D) $(x-3)(x^2+6x+9)$

6 The parametric equation of a function is:

$$x=2t^2,\ y=4-t.$$

The Cartesian equation is

(A) $x=4(2-y)^2$ (B) $x=2(y-4)^2$

(C) $x=2(y+4)^2$ (D) $x=2(4-y)^2$

7 Evaluate $\lim_{x\to 0}\frac{x}{\sin 2x}$:

(A) 0 (B) 0.5

(C) ∞ (D) 2

8 Which expression is equal to $\int \sin^2 3x \, dx$:

(A) $\frac{1}{2}\left(x - \frac{1}{3}\sin 3x\right) + C$

(B) $\frac{1}{2}\left(x + \frac{1}{3}\sin 3x\right) + C$

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9 A particle is moving in simple harmonic motion with displacement x. Its velocity v is given by

$$v^2 = 16(9 - x^2)$$

What is the amplitude, A, and the period, T, of the motion?

(A) $A = 3$ and $T = \frac{\pi}{2}$

(B) $A = 3$ and $T = \frac{\pi}{4}$

(C) $A = 4$ and $T = \frac{\pi}{3}$

(D) $A = 4$ and $T = \frac{2\pi}{3}$

10 The polynomial $P(x) = x^3 + ax^2 + ax + 1$ leaves a remainder of 3 when divided by $(x - 2)$.
The value of a is:

(A) 1

(B) -1

(C) -3

(D) 3

END OF SECTION 1

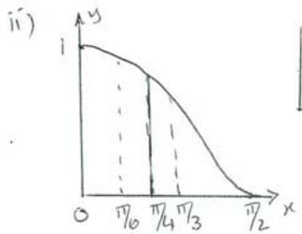
Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s) MULTIPLE CHOICE.	comments
1. (D) 2. (C) since for $y=\sin^{-1}x$, $-1 \le x \le 1$, $-\pi/2 \le y \le \pi/2$ 3. (B) 4. (C) 5. (C) 6. (D) 7. (B) 8. (C) 9. (A) 10. (B)	

QUESTION 11

Suggested solution(s)	comments
a) $\frac{d}{dx}(e^{x^2}) = e^{x^2}.2x$ 1	good attempt here.
$\frac{d^2}{dx^2}(e^{x^2}) = \frac{d}{dx}(e^{x^2}.2x)$ $= e^{x^2}.2 + 2x.e^{x^2}.2x$ $= 2e^{x^2}(1+2x^2)$ 1 (2)	poorly done by too many students. failure to recognise a product.
b) $\int_1^k (3 - \frac{1}{x^2})\,dx = 0$ $\therefore \left[3x + x^{-1}\right]_1^k = 0$ 1 $3k + \frac{1}{k} - (3+1) = 0$ $3k^2 + 1 - 4k = 0$ $3k^2 - 4k + 1 = 0$ 1 $(3k-1)(k-1) = 0$ $\therefore k = 1$ or $k = 1/3$ 1 (3)	– difficulty in dealing with k on the denominator

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s) QUESTION 11.	comments
c) Let $u = 1 + e^x$, $du = e^x dx$ 1 | when $x=0$, $u=2$; $x=\ln 2$, $u=3$ 1 $\therefore \int_0^{\ln 2} \frac{e^x}{(e^x+1)^2}\,dx = \int_2^3 \frac{du}{u^2}$ 1 $= \left[-\frac{1}{u}\right]_2^3$ $= -\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$. 1 (4)	Some issues with set up of parameters and du and dx.
d) i) $I = \int_0^{\pi/2} \cos^2 x\,dx$ where $\cos^2 x = \frac{1}{2}(\cos 2x + 1)$ $= \frac{1}{2}\int_0^{\pi/2} \cos 2x + 1\,dx$ 1 $= \frac{1}{2}\left[\frac{1}{2}\sin 2x + x\right]_0^{\pi/2}$ $= \frac{1}{2}\left[\frac{1}{2}\times 0 + \pi/2\right] = \pi/4$ 1 (2)	Failure to recognise the need for double angle formula. exact form!
ii) 	

x	0	π/4	π/2
$\cos^2 x$	1	1/2	0

Comment: Table of this form is very helpful.

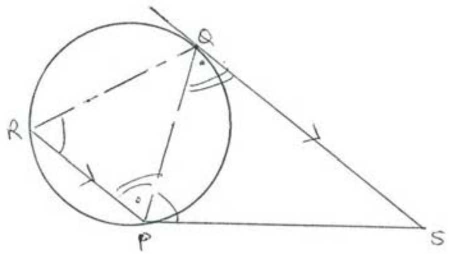
Suggested solution(s)	comments
$A \doteq \frac{h}{3}\left[d_F + 4d_M + d_L\right]$ 1 OR $\doteq \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$. $= \pi/12\left[1 + 4\left(\frac{1}{2}\right) + 0\right]$. $= 3\pi/12$ or $\pi/4$ 1 (2)	Some students need to learn the formula.

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s) QUESTION 11.	comments
e) i) $LHS = e^{x\ln 2}$ $= e^{\ln(2^x)}$ $= 2^x$ $= RHS$. 1	Need to learn index rules for e^x and $\ln x$.
ii) $\frac{d}{dx}(2^x) = \frac{d}{dx}(e^{x\ln 2})$ (2) $= e^{x\ln 2} . \ln 2$ $= 2^x . \ln 2$ 1	Follows from (i)

QUESTION 12.

a)



Suggested solution(s)	comments
i) Join PQ 1 (2) PS = QS (tangents from an external point are equal.) $\therefore \Delta PSQ$ is isosceles (two sides equal) 1	Need to learn def'n of tangents from an external pt. Identify what makes a triangle isosceles
ii) Join QR As ΔPSQ is isosceles, 1 $\angle PQS = \angle QPS$ (base angles of isosceles Δ are equal) $\angle PQS = \angle PRQ$ ($\angle$ between chord and tangent equals $\angle$ in alternate segment) 1 (2) $\angle SQP = \angle RPQ$ (equal alternate angles from // lines)	ditto definition needed reason

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s) QUESTION 12.	comments
$\therefore \Delta PQR$ is isosceles (two equal angles)	
iii) QP = QR (ΔPQR is isosceles; QP, QR are opposite equal angles) (1)	Explanation.
b) $\frac{dT}{dt} = -k(T-21)$ i) $T = 21 + Ae^{-kt}$ $\frac{dT}{dt} = 0 + A.e^{-kt}.-k$ (1) $= -k . Ae^{-kt}$ where $T-21 = Ae^{-kt}$ ie $\frac{dT}{dt} = -k(T-21)$.	Verification requiring the derivative and substitution.
ii) $t=0$, $T=27$ $27 = 21 + A \rightarrow \underline{A=6}$ 1 $t=1$, $T=25$ $25 = 21 + 6e^{-k}$ $\therefore 6e^{-k} = 4$ $\therefore e^{-k} = \frac{2}{3}$ (2) $\therefore -k = \ln\left(\frac{2}{3}\right)$ $\underline{k = \ln\left(\frac{3}{2}\right)}$ 1	Well done Well done.
iii) When $T = 37$, $37 = 21 + 6e^{-\ln\left(\frac{3}{2}\right)t}$ $16 = 6e^{-\ln\frac{3}{2}t}$ $\ln\left(\frac{16}{6}\right) = -\ln\frac{3}{2}t$ $t = \ln(16/6) \div -\ln\left(\frac{3}{2}\right) = -2h\ 25min$ $\therefore$ Time $= 3:27 - 2:25 = 1:02$ am	Not as well done. Do not write "calculator speak". Express values with hours and minutes.

Reasonably well done.

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s) QUESTION 12 continued.	comments
c) $2x^3 - x^2 - 5x + 6 = 0$.	
$\alpha+\beta+\gamma = \frac{-(-1)}{2}$ $= \frac{1}{2}$	– These need to be learned.
$\alpha\beta+\beta\gamma+\alpha\gamma = -\frac{5}{2}$	– ditto
$\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\alpha\gamma)$ $= (\frac{1}{2})^2 - 2(-\frac{5}{2})$ $= \frac{1}{4} + 5$ $= 5\frac{1}{4}$	– careful subst'n needed.
d) Prove: $\frac{1}{1\times2} + \frac{1}{2\times3} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$	
Test n=1. LHS $= \frac{1}{1(2)}$ RHS $= \frac{1}{2}$ LHS = RHS. ∴ true for n=1.	
Assume true for n=k. $\frac{1}{1\times2} + \frac{1}{2\times3} + \dots + \frac{1}{k(k+1)} = \frac{k}{k+1}$ →(A) Prove for n=k+1. ie prove $\frac{1}{1\times2} + \frac{1}{2\times3} + \dots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$	Some rocky presentations here.
LHS $= \frac{1}{1\times2} + \frac{1}{2\times3} + \dots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)}$ $= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$ $= \frac{k(k+2)+1}{(k+1)(k+2)} = \frac{(k^2+2k+1)}{(k+1)(k+2)}$ $= \frac{(k+1)^2}{(k+1)(k+2)}$	

$= \frac{k+1}{k+2}$ = RHS.

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
If true for n=k, then true for n=k+1 ∴ Statement is true for n=1, 2, 3, ... and all positive integral n.	
QUESTION 13.	2marks suggests 2 steps or 2 reasons 1, Stationary 2, Some comment about shape of cubic. Eg Increasing
a) i) $y = x^3 + 2x + 4$ $\frac{dy}{dx} = 3x^2 + 2 > 0$ for all x. ∴ No stationary pts and f(x) is always increasing. ① mark ∴ cubic will only have one real root α. ← ① mark	
ii) $P(-2) = (-2)^3 + 2(-2) + 4 = -8$ $P(-1) = (-1)^3 + 2(-1) + 4 = 1$ hence, $-2 < \alpha < -1$ ① mark as the function values alternate in sign.	Well done
iii) $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ where $x_0 = -1$. $x_1 = -1 - \frac{1}{5}$ ① mark substitution correct $= -1.2$ ① mark	Well done.

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s) QUESTION 13

b) $v^2 = 24 - 8x - 2x^2$

i) at rest $\rightarrow v = 0$ (1) mark

$2(12 - 4x - x^2) = 0$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$\therefore x = -6$ or $x = 2$ (1) mark

ii) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}(12 - 4x - x^2)$

ie $a = -4 - 2x$ (1) mark

iii) For max speed, $a = 0$

$\therefore -4 - 2x = 0$

$x = -2$ (1) mark

$v^2 = 24 + 16 - 8$

$= 32$

$\therefore v = \pm\sqrt{32} = \pm 4\sqrt{2}$

ie max speed is $4\sqrt{2}$ (1) mark

c) $x = 20t\cos\theta$ $\quad y = 20t\sin\theta - 5t^2 + 175$

when $t = 0$, $v = 20$, $\theta = 30°$ (1) mark

$\therefore x = 10\sqrt{3}t$ $\quad y = 10t - 5t^2 + 175$

i) when $y = 0$, $5t^2 - 10t - 175 = 0$ (1) mark

$5(t^2 - 2t - 35) = 0$

$5(t-7)(t+5) = 0$

$t = 7$ or $t = -5$

But, $t \geq 0$, $\therefore t = 7$ seconds. (1) mark

ii) $\dot{x} = 10\sqrt{3}$, $\dot{y} = 10 - 10t$ (1) mark

$v^2 = (\dot{x})^2 + (\dot{y})^2$

$= (10\sqrt{3})^2 + (-60)^2$

when $t = 7$, $\dot{y} = -60$

$= 300 + 3600$ (1) mark

$v^2 = 3900 \rightarrow v = 10\sqrt{39}$ m/s.

comments

b) i) Well done

ii) Some students unaware of $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

Some confused $\frac{dv}{dx}$ with $\frac{dv}{dt}$

c) Some students said at ground level $y = 175$, instead of 0 as shown in diagram & equation. Some did not obtain equations for $\dot{x}$ and $\dot{y}$ to enable use of $v^2 = \dot{x}^2 + \dot{y}^2$

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s) QUESTION 14.

a) i) $\cos x - \sqrt{3}\sin x \equiv R\cos(x + \alpha)$

$= R\cos x\cos\alpha - R\sin x\sin\alpha$.

$R^2 = 1 + (\sqrt{3})^2$

$= 4$

$R = 2$ (1) mark

$R\cos\alpha = 1$, $R\sin\alpha = \sqrt{3}$ $\}$ $\tan\alpha = \sqrt{3}$

$\therefore \alpha = \pi/3$ (1) Mark

(Since $0 \leq \alpha \leq \pi/2$)

ie. $\cos x - \sqrt{3}\sin x = 2\cos(x + \pi/3)$

ii) $2\cos(x + \pi/3) = 1$ (1) mark.

$\cos(x + \pi/3) = \frac{1}{2}$ and $0 \leq x \leq 2\pi$

So $\pi/3 \leq x + \pi/3 \leq 2\pi + \pi/3$.

$\therefore x + \pi/3 = \pi/3$ OR $5\pi/3$ or $7\pi/3$

$x = 0$ OR $4\pi/3$ or 2π. (1) mark

b) $x^2 = 4ay \rightarrow y = \frac{x^2}{4a}$, $y' = \frac{2x}{4a}$

i) at P, $x = 2ap$, $y' = \frac{4ap}{4a} = p$. (1) mark

Eqn of tangent at P is

$y - ap^2 = p(x - 2ap)$ (1) mark

$y - ap^2 = px - 2ap^2$

$\therefore px - y - ap^2 = 0$.

ii) $X(\_\_, 0)$. Put $y = 0$.

$\therefore px = ap^2$

$x = ap$

ie X is $(ap, 0)$ (1) mark

comments

a) i) Well done.

ii) Most students did not expand the domain and lost 2π solution

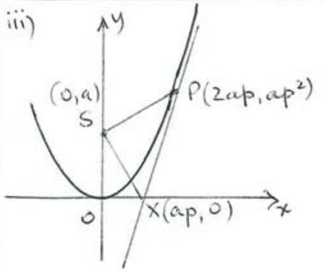
b) Well done by those who attempted it.

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s) QUESTION 14 continued.

comments

iii)



$m(PX) = \frac{ap^2 - 0}{2ap - ap}$

$= p$

$m(SX) = \frac{a - 0}{0 - ap}$

$= \frac{a}{-ap}$

$= -\frac{1}{p}$ (1) mark

Since $p \times -\frac{1}{p} = -1$, $PX \perp SX$ (1) mark

iv) PS is diameter as $\angle PXS = 90^\circ$ (angle in semicircle) (1) mark

$\therefore$ Centre of circle C = coords of midpoint PS

$= \left(\frac{2ap}{2}, \frac{ap^2 + a}{2}\right)$ (1) mark

$\therefore C = \left(ap, \frac{a(p^2+1)}{2}\right)$

Comments: Some students did not realise PS must be diameter. Not many attempts made at this easy question.

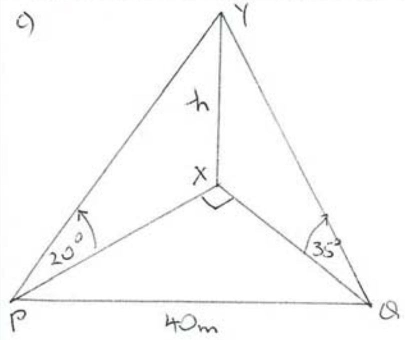
THGS Exam Feedback

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s) QUESTION 14 c)

comments

c)



i) In ΔPXY, $\frac{h}{PX} = \tan 20^\circ$

$\therefore PX = \frac{h}{\tan 20^\circ}$ (1) mark

ii) In ΔQXY, $\frac{h}{QX} = \tan 35^\circ$

$QX = \frac{h}{\tan 35^\circ}$ (1) mark

iii) $\angle PXQ = 90^\circ$.

$PX^2 + QX^2 = 40^2$

$\frac{h^2}{\tan^2 20^\circ} + \frac{h^2}{\tan^2 35^\circ} = 1600$ (1) mark

$h^2\left(\frac{1}{\tan^2 20^\circ} + \frac{1}{\tan^2 35^\circ}\right) = 1600$

$h = \frac{\sqrt{1600}}{\sqrt{\left(\frac{1}{\tan^2 20^\circ} + \frac{1}{\tan^2 35^\circ}\right)}}$

$= \frac{40}{\sqrt{\frac{1}{\tan^2 20^\circ} + \frac{1}{\tan^2 35^\circ}}}$

$= \frac{40}{\sqrt{9.5882389}}$

$= 12.91785..$

$h \doteqdot 13$ metres (1) mark